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Knowledge of affine and quadratic functions manifested by mathematics undergraduate students[1]

Conhecimentos de funções afim e quadrática manifestados por estudantes de licenciatura em matemática[1]

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Abstract
The research presented here aimed to analyze the knowledge of students in the 4th year of the Mathematics Course about the concept of function. For its development, two mathematical tasks were developed, articulating affine and quadratic functions, which were implemented remotely and synchronously for 13 students. The analyzes were carried out based on the Theory of Conceptual Fields, in order to understand the resolutions and knowledge expressed by the students. Data analysis shows students mistakes when describing the graphs in natural language; errors associated with algebraic operations; and difficulty in presenting the functions corresponding to graphs, especially of quadratic functions. Difficulties such as these show that the concept of function is not yet well established by some of these future mathematics teachers.
Keywords: Mathematics Education. Conceptual Fields Theory. Higher Education.

Resumo
A pesquisa aqui apresentada visou analisar conhecimentos de estudantes do 4º ano do Curso de Matemática acerca do conceito de função. Para o seu desenvolvimento foram elaboradas duas tarefas matemáticas, articulando funções afim e quadrática, que foram implementadas de maneira remota e síncrona para 13 estudantes. As análises foram realizadas com base na Teoria dos Campos Conceituais, com vista a compreender as resoluções e conhecimentos manifestados pelos estudantes. A análise dos dados mostra equívocos dos estudantes ao realizar a descrição dos gráficos em linguagem natural; erros associados a operações algébricas; e dificuldade para apresentar as funções correspondentes aos gráficos, especialmente de funções quadráticas. Dificuldades como estas mostram que o conceito de função ainda não está bem estabelecido por alguns destes futuros professores de matemática.
Palavras-chave: Educação Matemática. Teoria dos Campos Conceituais. Ensino Superior.

Introduction

The concept of function, as well as the notions of variable, dependence, correspondence, regularity and generalization – which are the basis for understanding this concept – are part of situations that can be presented to students from the Early Years of Elementary School (CALADO, 2020; MIRANDA, 2019). These situations may involve problem solving and direct proportional variations between two quantities (BRASIL, 2018).

Officially, the concept of function should be studied in the 9th year of Elementary School, and further in High School when exploring situations that "[...] allow the representation, in a Cartesian coordinate system, of magnitude variations, in addition to the analysis and characterization of the behavior of this variation (directly proportional, inversely proportional or non-proportional)" (BRASIL, 2018, p. 530).

Historically, education on functions has privileged the exploration of their algebraic scope, seeking to present generality, introducing techniques or algorithms (CAMPITELI; CAMPITELI, 2006). Regarding the concept of affine functions, for example, Miranda (2019) shows that the situations presented in Mathematics textbooks for Elementary and High Schools have very little difference in their structure. However, in order to understand a concept, Vergnaud (2009a, 1993) defends the need to diversify situations so that during the education process the concept can be grasped by subjects, based on different situations experienced by them.

For this study, we were guided by the Theory of Conceptual Fields (TCF), in which Vergnaud (1996; 2009b) suggests that a concept is understood by students not in isolation, but from a diversity of situations that encompass several other concepts, theorems, properties, symbols, representations, operative invariants, etc., which are interconnected in what the researcher calls the Conceptual Field. Furthermore, according to Vergnaud (2009a), a concept is grasped by subjects when it simultaneously mobilizes the operative and predicative forms of knowledge, which are associated, respectively, with knowing how to do something and knowing how to make objects and their properties explicit.

The interest in analyzing the students' knowledge about affine and quadratic functions is based on studies carried out by the Study and Research Group on Mathematics Didactics – GEPeDiMa[2], of which the authors of this paper are part. One of the Group's objectives is "[...] to map the process of construction of the concept of function, seeking to identify knowledge mobilized by subjects of different ages when solving problem situations" (NOGUEIRA; REZENDE, 2019, p. 196) referring to this concept. The present study is the result of a, undergraduate research that culminated in the first author's undergraduate thesis, with academic advisory by the second author, coming to add to the research and results found by GEPeDiMa.

With this in mind, we established the general objective of this research: to analyze knowledge manifested by undergraduate mathematics students when solving tasks involving affine and quadratic functions. In this sense, we sought to analyze their explicit and implicit mathematical knowledge, whether suitable or not, such as: resolution strategies and symbolic representations, and the operative and predicative forms of knowledge, manifested by the research subjects during the resolution of tasks.

In the next sections we present the theoretical framework that supports the development of this research, followed by the methodological procedures, data analysis and final considerations.

Some aspects of the Theory of Conceptual Fields

The Theory of Conceptual Fields (TCF) was developed in the 1980s by the French psychologist Gérard Vergnaud. It is a cognitivist theory that brings contributions to the Didactics of Mathematics, based on the work by Jean Piaget (1896-1980) and Lev Vygotsky (1896-1934).

The TCF aims to understand "[...] the specific developmental problems within the same field of knowledge" (VERGNAUD, 1996, p. 11), and to establish a structure that makes it possible to understand the affiliations and the ruptures of previous ideas between different types of knowledge (VERGNAUD, 1993).

According to Vergnaud (1993), a subject, when acquiring new skills and understanding a new concept, does so progressively, through experiences that they have over several years during their schooling. These experiences are related or even derived from practical and theoretical situations. Vergnaud (2009b) states that the knowledge that a child acquires must be constructed "[...] in direct relationship with the operations that they, as a child, are capable of carrying out in reality, with the relationships that they are capable of discerning, composing and transforming with the concepts that they are progressively building" (VERGNAUD, 2009b, p. 15).

These situations and experiences lead us to what Vergnaud calls the Conceptual Field, "[...] a set that is vast, however, it is organized from a set of situations" (VERGNAUD, 2003, p. 30). For the researcher, students can understand a concept through the various situations they have experienced over time, which demand other interconnected elements, such as theorems, properties, symbols, representations, among others that together make up what Vergnaud calls the Conceptual Field (VERGNAUD, 2009b; CALADO, 2020).

Considering the elements of this theory, a set of schemes and symbolic representations is necessary for the interpretation of these situations to be possible (VERGNAUD, 2003). Schemes are described by the author as "[...] a way of organizing the task, intended for a class of situations" (VERGNAUD, 2019, p. 7) that includes one or more objectives, action rules, operative invariants, and possibilities of inference.

Operative invariants can be of two logical types – theorems in action and concepts in action. Vergnaud clarifies that concepts in action are always true, and it is only possible to classify them according to their relevance to the situation presented. They are different from theorems in action, which can be classified as true or false, as they are propositions that interfere in the development of the task. Thus, a theorem in action, even if false, remains a theorem in action (VERGNAUD, 1993, 2019).

As a way of exemplifying theorems in action and concepts in action, Vergnaud (1993) presents the following example: if we multiply a number of objects sold by 2, 3, 4, 10 or 100, the price paid will be 2 , 3, 4, 10 or 100 times greater, and this knowledge can be expressed by the true theorem in action $f(nx) = nf(x)$. For all n in this case, the concept in action employed is distributivity.

Vergnaud (1993) presents a definition for the term concept from a perspective based on psychology. In this context, a concept is determined by the trio (S, I, L): S, called reference, is the set of situations that give meaning to the concept, which require the mastery of a variety of concepts, schemes and inter- related symbolic representations; I, called meaning, refers to the set of concepts that contribute to understanding situations – they are the operational invariants manifested in the schemes, in the organization, development and resolution of situations by the subjects; and L, the signifier, the linguistic and symbolic forms that allow expressing objects of thought and explicit or non-explicit concepts within situations (VERGNAUD, 2009a).

Specifically referring to the set of signifiers for the concept of function, there are several symbols and representations intertwined with this concept such as graphic representations in Cartesian plane, algebraic representations, numerical representations, representations in natural language and tabular representation, in addition to of other symbolic representations necessary for the construction and interpretation of graphs, algebraic and numerical solutions.

In this sense, based on the Theory of Conceptual Fields, we argue that a subject understands a concept by knowing how to deal with and resolve a set of situations, that is, by manifesting organized schemes for their resolution. These situations demand properties, theorems and invariants for their resolution, as well as different symbolic representations.

For the author of the TCF, the definition of a situation is close to that of a task, so that "[...] every complex situation is analyzed as a combination of tasks, whose specific nature and difficulties must be well known" (VERGNAUD, 2003, p. 9). Vergnaud (1993) presents two ideas related to situations: the first refers to variety, a concept can be linked to different situations, that is, a Conceptual Field encompasses a large class of situations; the second refers to history, interpreted as the students' experiences, so that the students' education happens through several situations experienced by them (VERGNAUD, 1993).

Proposing different situations that vary not only in context and the numerical data, but that alternate the very structure of the question in their problems, is fundamental according to Vergnaud, since most students' experiences do not cover a wide variety of problems (REZENDE, BORGES, 2017).

In line with this diversity of situations, Vergnaud (2003) emphasizes the importance of proposing situations that, when developed by students, allow them to assess both their competence in doing, represented by the operative form of knowledge, and knowing how to explain, represented by the predicative form of knowledge. Thus, Vergnaud (2008b, p. 01) points out that "[...] academic and professional learning both concern knowledge, its operative form and its predicative form", and that the "[...] passage, from an operative form of knowledge to a predicative form [...] is one of the biggest challenges in education" (VERGNAUD, 2008a, p. 17-18).

Although the operative form allows us to act on a situation in more complex and refined ways than the predicative form, it is the predicative form of knowledge that allows us to state what has been accomplished and represent it symbolically (VERGNAUD, 2002, 2011).

Thus, the predicative form of knowledge is fundamental in the construction of a concept, so that the "[...] invariance of the symbolic form comes to the aid of the invariance of concepts" (VERGNAUD, 2002, p. 15), and "[...] the symbol mainly allows us to speak of objects and properties that are not accessible to direct perception" (VERGNAUD, 2002, p. 15).

In the next section we describe the methodological procedures, presenting the articulation between the Theory of Conceptual Fields and the objective of the study.

Methodological procedures

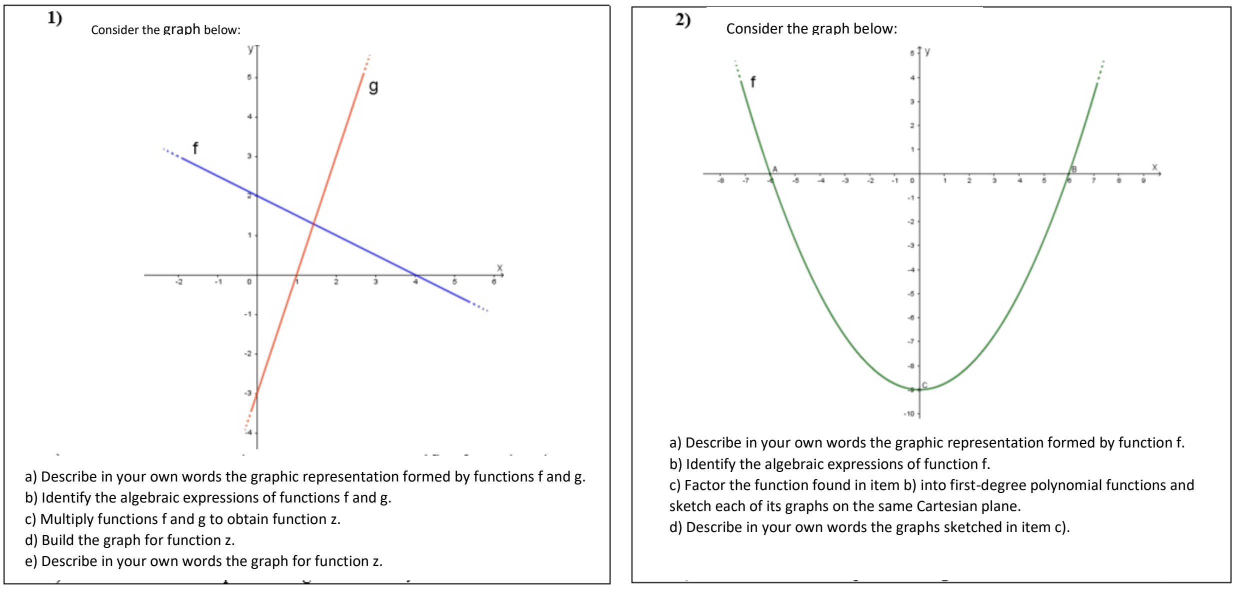
This research aimed to analyze the knowledge manifested by undergraduate mathematics students when solving tasks involving affine and quadratic functions. The collaborating subjects of the research were 13 students of the 4th year of a Mathematics degree course at a public institution in the state of Paraná. Participants on this level of education were chosen because studies (NUNES; SANTANA, 2017; PIRES; MERLINE; MAGINA, 2015) have shown that higher education students, and even professors, express misunderstandings about functions. The choice of the University was due to the proximity and bond of the researchers to the institution.

For the data collection and production of research data, we invited the students to a remote meeting via the Google Meet platform. We asked them to solve the proposed tasks individually and synchronously within a period of 3 class hours. When completing the tasks, the resolutions were photographed and sent to the research proponents by messaging applications or via email.

The research instrument is composed of two tasks developed considering as a starting point the ideas presented in the dissertation by Llanos (2012), defended at the Universidad Nacional de Centro de la Provincia de Buenos Aires. For the present research, we adapted and expanded one of the tasks proposed by Llanos (2012), so that, given the different representations about affine and quadratic functions, both the predicative form of knowledge and the operative form of knowledge could be manifested by the students during their resolutions. Among the various tasks developed by Llanos (2012), we chose to adapt "*Situation 1*," developed by the researcher, in which students were asked to identify positive and negative function intervals based on graphs of affine and quadratic functions – roots, axes of symmetry, vertices, among other aspects.

The adapted task was presented to the members of GEPeDiMa and after discussions it was proposed that the task be re-developed considering the application conditions of this research. Specifically, the time required to solve the tasks proposed by Llanos (2012) was considered, compared to the time that we would have available for the students collaborating in this research. The proposed tasks are shown in Figure 1:

Figure 1 - Tasks 1 and 2



Source: the authors.

Considering the TCF, each item of each task was carefully developed considering the different situations to be proposed for the students to solve: description of the graphic representation; identification of algebraic expressions; multiplication or factorization of functions; graphics construction. Each of these situations demands different representations: Cartesian graph, algebraic representation, numerical representation, natural language; in addition to several symbols necessary for their resolution. Also, these tasks have the potential to articulate affine and quadratic functions, a fact that is not always contemplated in Mathematics classes and textbooks in Elementary Education.

In the items in which the student must look for for the functions associated with the graphs (1b and 2b) and in those in which algebraic operations are performed (1c and 2c) it is expected that the operative form of knowledge will be manifested by students. In the items in which students are asked to describe the graphs constructed by them (1e and 2d), we expected that the predicative form of knowledge would be manifested, that is, the description in natural language[3] of the graphics constructed by the students. We clarify that, although items (1a and 2a) require students to interpret the graph presented in natural language, we do not assume the possibility of manifesting the predicative form of knowledge, because the graphs in question were not constructed by the students, although these are essential matters for the analysis of graphical interpretations.

Among the limiting factors for the implementation of the tasks, we highlight the Covid-19 pandemic, which occurred during the period of research. Due to the pandemic, throughout the data collection period, in-person academic activities were suspended in several schools and universities. For this reason, data collection took place remotely and synchronously, being carried out during class hours.

The analyses were carried out based on the TCF, seeking to identify the symbolic representations and resolution strategies employed by participants, whether correct or not. Also based on the TCF, we turned our attention to the explicit misconceptions and, possibly, the implicit ones, manifested by the students.

Data analysis

As a way of organizing the analyses, we approached each item of the tasks according to the objective with which it was developed. Therefore, we looked at the students' strategies, the description in natural language of the graphs, the presentation of the function associated with the graph, the algebraic operations with the established functions and the construction of graphs.

In order to name the students preserving their anonymity, they were identified according to the order in which they sent in the protocols in numerical sequence, between A1 and A13. The resolutions presented by the students were listed as adequate, partially adequate, and inadequate, considering the development presented by the student. As the resolution of task items are dependent on each other, task items will be analyzed individually. We chose to proceed with the analysis in this way, as there are cases in which, although the final answer is different from the expected answer, this may be the result of errors in previous items and not errors in that specific item.

We considered as adequate answers those that met what was proposed in the statement, according to what was expected for students in the 4th year of the Mathematics undergraduate course. We interpreted as partially adequate ones, those answers that, although they are not divergent from the expected answer, are incomplete. Inadequate answers did not respond to what was proposed in the statement. And finally, tasks not performed were those in which the student did not present any mathematical development or argumentation.

Table 1 presents a summary of the distribution of the answers presented by the students, as well as the types of errors found.

Table 1 - Distribution of answers presented by students

<table>
<tr><th></th><th>Adequate</th><th>Partially adequate</th><th>Inadequate</th><th>Not performed</th><th>Types of errors identified and in which tasks they were manifested</th></tr>
<tr><td>1a</td><td>3</td><td>9</td><td>1</td><td>0</td><td rowspan="9">• Description of graphs with no mathematical rigor (1a, 1e, 2a and 2d)
• Incorrect naming of elements in the function and its graph (1a, 1e, 2a and 2d)
• Use of techniques that do not correspond to the task (1b and 1c)
• Errors in algebraic calculations (1c and 2b)
• Presenting a graph that does not correspond to the function (1d)
• Answers with no mathematical argumentation (2b and 2c)</td></tr>
<tr><td>1b</td><td>8</td><td>4</td><td>1</td><td>0</td></tr>
<tr><td>1c</td><td>9</td><td>1</td><td>3</td><td>0</td></tr>
<tr><td>1d</td><td>11</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>1e</td><td>9</td><td>3</td><td>0</td><td>1</td></tr>
<tr><td>2a</td><td>1</td><td>12</td><td>0</td><td>0</td></tr>
<tr><td>2b</td><td>5</td><td>3</td><td>4</td><td>1</td></tr>
<tr><td>2c</td><td>2</td><td>3</td><td>2</td><td>6</td></tr>
<tr><td>2d</td><td>1</td><td>5</td><td>1</td><td>6</td></tr>
</table>

Source: Research data (2021).

First, we will turn our attention to items 1a and 2a of the tasks, which asked for the description of the graphs given in the statements. As adequate answers, we expected that, using mathematical rigor, at least the intercepts with the x and y axes, vertices, concavities, growth or decrease would be described and that the graphs of the functions would be identified as straight lines, parabolas or curves.

We found that three (03) students answered item 1a correctly and one (01) answered item 2a correctly. As an example of an adequate description, we bring the resolution of student A2, in which he states: "*It is a parabola with concavity facing upwards, with roots* $x = -6$ *and* $x = 6$, $x_v = 0$, $y_v = -9$". In it, the student points out the behavior of the parabola, its roots and its vertex correctly.

As an example of a partially adequate resolution regarding the description of the graphs given in the statement, we present the resolutions by students A6 and A7. Respectively, the students say, "*Two distinct lines with a single intercept*" and "*The graph is a* *parabola*." In these examples we note that although the statements presented by the students are correct, elements such as roots, intercepts and the behavior of the lines or parabola were not explained. We identified nine (09) partially adequate resolutions in item 1a, and twelve (12) in item 2a.

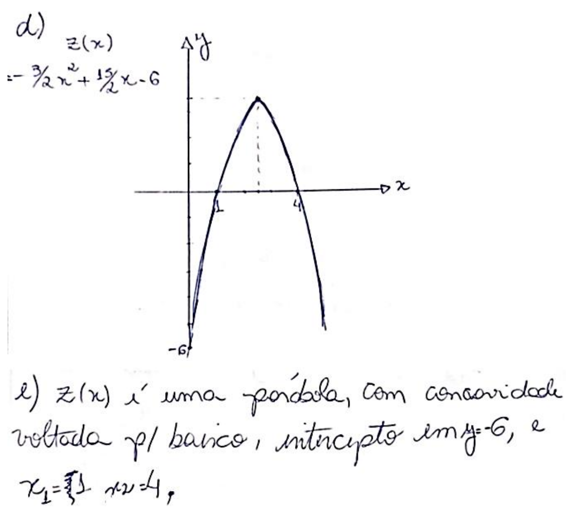
We also listed as partially adequate resolutions those in which students did not show mathematical rigor when describing the graph, using terms that were not mathematically adequate. In these resolutions, phrases such as "*growing parabola*," "*positive parabola*" emerged. There were also cases of inversion of the "x axis" by "x plane," among other terms that are not mathematically adequate.

Only student A10 presented an inadequate resolution for items 1a and 2a, giving as an answer to item 1a the question: "*An x?*" When observing the student's protocol, we noticed that he guided himself by elements of the graph to solve the following items of the task, which possibly indicates a lack of understanding of what was requested in the statement, an inference that is reinforced by what was presented by the same student in item 2a in which he described the graph as "A parabola," a description that, although simple, is partially adequate, as it only omits other aspects of the graph.

Still on the description of the graphs, but now specifically about items 1e and 2d, which demanded the description of the graphs that were built by the students during the development of the tasks, we noticed a difference from the descriptions given in items 1a and 2a.

One of these differences can be observed in items 1a and 1e. Item 1a presented three (03) adequate resolutions and nine (09) partially adequate, while item 1e, which asked for the description of a graph constructed during the task, presented nine (09) adequate resolutions and three (03) partially adequate ones, so that in item 1e the students presented and described more elements of the graph correctly. As an example, let us look at the answer given by student A5. In the description made in item 1a of a graph given in the statement, the student presents as an answer "*f and g form two straight lines, one increasing and one decreasing*"; comparing this description with the description given in a graph constructed by the student himself in item 1e, shown in Figure 1, we can see that the student made a more detailed description when he was based on his own construction.

Figure 1 - Resolution presented by student A5 for items 1d and 1e



Source: Research data (2021).

Although we identified a higher rate of adequate resolutions in items 1e and 2d when compared to items 1a and 2a, we observed some conceptual errors, such as the case of student A3 who, despite having presented an adequate resolution for item 1a, described a quadratic function as a "descending function" in item 1e, which is incorrect, in addition to omitting elements present in the graph.

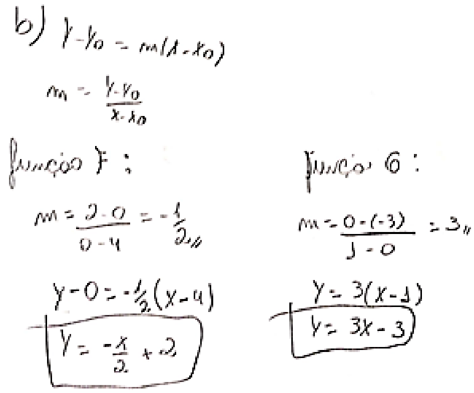
This lack of mathematical rigor or the use of terms that are not mathematically adequate is more evident when we look at the descriptions made in item 2d. In this item we identified five (05) partially adequate resolutions and one inadequate resolution. Difficulties in graph interpretation, similar to these, are also presented by Nunes and Santana (2017), when they highlight students' difficulties in "[...] identifying and relating the terms, perhaps because they only have an intuitive notion of the relationship between sets" (NUNES; SANTANA, 2017, p. 69). Difficulties like these reinforce the hypothesis that concepts related to functions may not be well established among some of the research subjects.

Still analyzing the difficulty faced by these students when performing the description of a graph, we point to the fact that, although five (05) students presented the expected function for item 2b, only one (01) gave an adequate description of the graph constructed in item 2d, pointing out a disparity between these students' ability to know how to do and to know how to explain.

Regarding items 1b and 2b of the tasks, which asked students to use the function associated with the graph given in the statement, the students resorted to different strategies to solve these items. Among the strategies adopted that resulted in correct solutions, we mention the algebraic manipulations of equations $ax + b = 0$ e $ax^2 + bx + c = 0$, the use of a reduced equation of the line $y = mx + n$, the use of geometry software, and trial and error.

Among the thirteen (13) students, eight (08) correctly answered item 1b and 5 correctly answered item 2b. Figure 2 below presents an example of a solution in which the reduced equation of the line was used to solve the item.

Figure 2 - Resolution presented by student A9 for item 1b



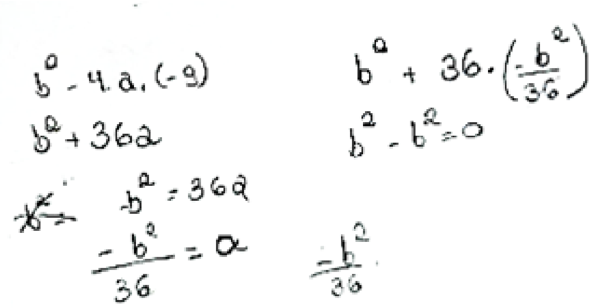
Source: Research data (2021).

Among the partially adequate solutions for items 1b and 2b, we identified two situations, the first associated with item 1b in which students found only one of the functions correctly, and the second related to item 2b, in which students only presented the final answer, with no mathematical argumentation.

Among the thirteen (13) students, five (05) had difficulties in presenting the function associated with the graph given in the statement, especially in item 2b, in which four (04) students gave inadequate responses and one (01) did not respond at all. We also highlight another four (04) students who presented the correct function for item 2b, but they did it without any development associated with that answer. Based on Vergnaud (2009a; 2009b), we infer that the subjects who collaborated in this research did not have ready schemes to start from the graphic representation of a function and present its algebraic expression; that is, the proposed tasks, items 1b and 2b, are characterized as a new situation for the students, which they did not have organized schemes to solve, so they had to look for different strategies to present a solution, which in most cases was not adequate.

As an example of the difficulties expressed by the students, Figure 3 presents a fragment of student A6's resolution of item 2b.

Figure 3 - Fragment of the resolution given by student A6 for item 2b



Source: Research data (2021)

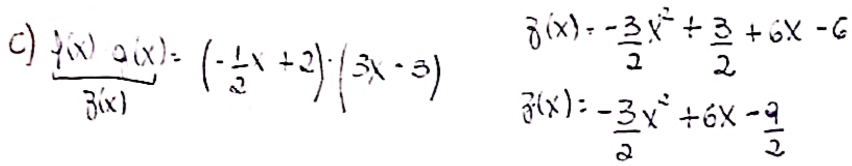
In the resolution, the student sought to trigger a scheme in the form of an algebraic algorithm to find the quadratic function associated with the graph of the statement. This is a valid resolution strategy, but he used the wrong knowledge, substituting a variable found in the same equation used to find it. Analyzing the development presented, we notice that, although it has determined the terms a and c correctly, the student sought to replace this term in equation $\Delta = b^2 - 4ac$, the same equation used to determine term a, while in this case it would be appropriate to replace a and c in equation $ax^2 + bx + c = 0$. The difficulties mentioned here are also pointed out by Nunes and Santana (2017).

Regarding the algebraic operations from the functions identified by the subjects, requested in items 1c and 2c of the tasks, in item 1c nine (09) students gave adequate answers, in which all performed the multiplication between the functions correctly. In item 2c, only two (02) students presented adequate answers, both using special products to obtain the factored form of the quadratic function.

As for the inadequate solutions given for items 1c and 2c, specifically regarding item 1c, three (03) students presented incorrect solutions. In this item, errors such as incorrect sums and cases in which the student only multiplied the numerical part of the term, not operating the unknown quantity, were observed.

Two examples of this class of errors follow in Figure 4.

Figure 4 - Resolution presented by student A2 for item 1c

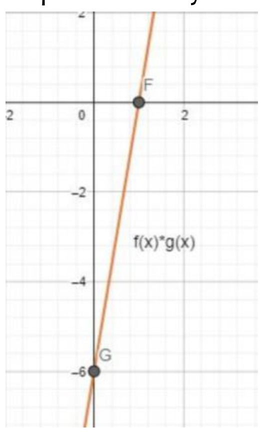


Source: Research data (2021)

We found other resolutions similar to the one given, in which it is possible to notice that in the multiplication between the affine functions, student A2, when performing the multiplication of the terms $-\frac{1}{2}x$ e -3, possibly forgot to consider the variable x, resulting in an algebraic mistake. Although they are 4th year students of the BA and Teaching Licensure in Mathematics, errors similar to this one have been found by Burigato and Bittar (2008) when working with eighth grade students, and by Cury and Cassol (2004) when looking at students who made mistakes when performing operations such as distributive ones, or errors when writing a term or notation.

Finally, regarding the construction of the graphs, among the thirteen (13) students, only one student presented an inadequate response regarding the construction of the graph, presenting a graph different from the function found by him in item 1d. Student A7 resorted to the use of GeoGebra to construct the graph, as shown in Figure 5. Although he presented adequate resolutions in the previous items, finding the quadratic function that should actually be sketched, he possibly made a mistake when inserting it in the software, not including exponent 2 in the term ax^2.

Figure 5 - Resolution presented by student A7 for item 1d



Source: Research data (2021).

We observed that the student did not identify that this construction does not represent a quadratic function, but an affine function. Considering that our analyses were based on the protocols given by the students, we can only infer possible situations that led to the student not identifying this inconsistency. Thus, the error presented can either be the result of something occasional and non-systemic, linked to a moment of inattention, or it can reflect a serious gap in training from when he was in school, for not realizing that the multiplication between two affine functions results in a quadratic function, a gap that may represent the possibility that he has not experienced situations like this during his schooling process, as argued by Vergnaud (2009a; 2009b).

Regarding item 2d, six (06) students did not construct of the graph, due to not having solved previous items. This gives us pause about the concepts of function and notions of algebra that are possibly not well established among these six (06) students. The fact that these students did not present the function demanded by the statement, even though they had the possibility of using resources such as GeoGebra, reinforces the idea that this was a new situation for them, a situation which these students did not have schemes established to solve.

Final remarks

This study sought to analyze the knowledge manifested by undergraduate mathematics students while solving tasks on quadratic and related functions, guided by the Theory of Conceptual Fields. To achieve this objective, two mathematical tasks were developed and implemented with 4th year students of an undergraduate mathematics course, future teachers, from a state university in the center-west of Paraná.

As for the solutions presented by the students and their strategies, we noticed mistakes, which were grouped in: i) description of the graphs, not explaining several elements arranged in the graphs; ii) difficulty in presenting the functions corresponding to the graphs; and iii) algebraic mistakes.

As for the descriptions of the graphs made by the students, associated here with the operative form of knowledge, we can make some inferences. The difference presented by the students when describing graphs given by the statements and those that were built by them was noticeable. However, analyzing the inadequate and partially adequate resolutions, we observed some conceptual errors and the use of mathematically inappropriate terms, so that six (06) students had difficulties when describing the graphs, possibly because it was an unusual or new situation to them.

For Vergnaud (1996; 2008a), the passage from the operative form of knowledge to the predicative form of a given object is one of the biggest challenges in education, and students are not prepared to carry out this type of "explicit work." Vergnaud (2008b, 2002) also states that learning a concept concerns both its operative form and its predicative form, which must be manifested jointly, something that we cannot say that occurred among these six (06) students mentioned.

The second point we looked at was the errors and difficulties shown by students to, starting from a graph, draw the function that is associated with that graph, including algebraic mistakes. Among the thirteen (13) students, five (05) expressed difficulties in finding the function associated with the graph given by the statement, and among the other eight (08) students, four (04) presented the function without mathematical reasoning. This difficulty allows us to affirm that at least five (05) participants of this research did not have ready schemes to, starting from a graphic representation, enunciate their algebraic expression, characterizing it as a new situation for them.

Students not having ready-made schemes for this situation, especially as we are dealing with future mathematics teachers, is noteworthy, as it indicates that throughout their schooling, in Elementary Education and in their undergraduate studies, they did not have, or had little, opportunities to solve tasks such as those discussed in this study. Based on Vergnaud (2009a; 2009b), this lack of varied situations throughout these students' training process, and the errors and difficulties manifested by them, indicates that the concept of function is not well established for them.

Thus, the research results show that the concept of function, which must be officially studied since the 9th year of Elementary School, furthered during high school and resumed during the Mathematics undergraduate course, is not a concept that is simple for students to understand. It is, in fact, a topic that generates difficulties and misunderstandings even on the part of students who are future Mathematics teachers.

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Notes

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[2] GEPeDiMa: https://prpgem.wixsite.com/gepedima
[3] In this research, we understand written Portuguese as natural language.

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